Math 1320: Solving Word Problems

Why is solving word problems important? Want to make a recipe for a dinner party? Well, to have enough for everyone you'll probably have to double, or maybe even triple the recipe. You may not actually write it out, but you're using a mathematical model to make the dish! Do you like to get coffee from your favorite coffee shop? When you budget for these visits, how many coffees can you buy each week? What size? You can create a model to answer these questions and many others. Math is most useful when applied to our everyday lives, not as random equations that you may practice solving in class.

What types of word problems may I be asked? We will first focus on mathematical models that use linear equations to solve word problems. Many of the problems we will encounter in class will deal with finance.

When we are given a word problem, it is written in English. We need to think of math as its own language, and in order to create the mathematical models, we need to translate from English to math. When reading a word problem, consider the steps in the table below:

Step 1	Read the problem carefully a few times. Underline, circle, or make a list of what is given in the problem and what the problem is asking you to look for. Assign x (Or any variable) to represent one of the unknowns in the problem.
Step 2	If needed, write expressions for any other unknown quantities in the problem in terms of x .
Step 3	Write an equation that models the information of the problem, in terms of x .
Step 4	Solve the equation and answer the problem's question.
Step 5	Check your solution in both the equation you wrote in step 3 and in the original wording of the problem.

Example 1. You are choosing between two gyms. One gym offers membership for a fee of $50 plus a monthly fee of $20. The other offers membership for a fee of $5 plus a monthly fee of $35. After how many months will the total cost at each gym be the same? What will be the total cost for each gym?

1. **Step 1:** After reading the problem, here is what I know:

Gym 1	Gym 2	Unknown
• Initial cost: $50 • Monthly fee: $20	• Initial cost: $5 • Monthly fee: $35	• How many months (let months $= x$)

2. **Step 2:** We need to know how much each gym will cost after a certain amount of months. Let's write an expression for each gym in terms of the unknown (x):

(a) **Gym 1:** $50 is a one-time cost, but we pay $20 each month (x), let $50 + 20x$ = the total cost for membership at Gym 1.

(b) **Gym 2:** $5 is a one-time cost, but we pay $35 each month (x), let $5 + 35x$ = the total cost for membership at Gym 2.

3. **Step 3:** We are asked to find how many months until the cost is the same (or equal). We need to set our expressions from step 2 equal to each other:

$$50 + 20x = 5 + 35x$$

4. **Step 4:** Solve the equation from step 4:

$$50 + 20x = 5 + 35x \qquad \text{This is the equation from step 4}$$
$$50 = 5 + 15x \qquad \text{Subtract } 20x \text{ from both sides}$$
$$45 = 15x \qquad \text{Subtract 5 from both sides}$$
$$3 = x \qquad \text{Divide both sides by 15}$$

Now let's find the cost at 3 months:

$$50 + 20x = 5 + 35x \qquad \text{This is the equation from step 4}$$
$$50 + 20(3) = 5 + 35(3) \qquad \text{Replace } x \text{ with 3}$$
$$50 + 60 = 5 + 105 \qquad \text{Multiply}$$
$$110 = 110 \qquad \text{Combine like terms}$$

The cost will be the same after 3 months and both gyms will cost $110.

5. **Step 5:** To check our solution, let's replace x with 3 in our original equation (from step 3). We did this in step 4 to find the cost of the two gyms after 3 months. In the original problem, we are asked for the costs of the gym to be equal, are they? Yes!

In the following problems, identify the unknown and assign it a variable:

1. You order coffee for $4 and a donut for $1.50 each time you visit your favorite coffee shop. Last week you spent $27.50 total on coffee and donuts. How many times did you visit the coffee shop last week? [Answer: x = visits to the coffee shop]

2. Amazon is having a sale on board games. After a 25% price reduction, you purchase a board game for $16. What was the price of the board game before the reduction?

   [Answer: x = original price of the board game]

3. The length of a rectangular swimming pool is 75 feet more than the width. If the perimeter of the pool is 360 feet, what are its dimensions? [Answer: x = the width of the pool]

For the previous problems, write expressions to represent the other unknown quantities in terms of x. Answers are below.

1. $4x$ = total amount spent on coffee $1.50x$ = total amount spent on donuts

2. There are no other unknown quantities (skip step 2 in this case)

3. $x + 75$ = the length